Learning Trajectories of Early Math

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1

Vision

- 4- and 5-year-olds
- Puzzle
- Cory puts 4 triangles together to make squares

2

Vision

Cory makes a new shape: A unit of units!

Another boy sees the square structure, but builds the wrong square

3

Vision

Finishing, Cory shows adult, who asks:
"How many triangles did you use?"

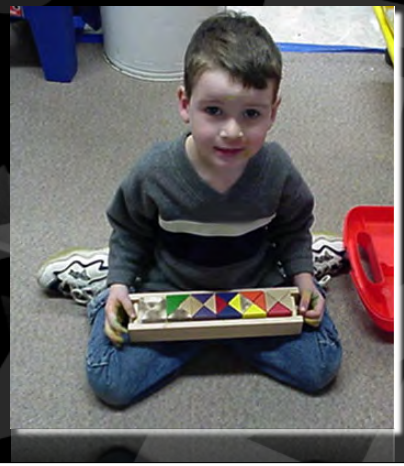
Cory counts: "24"

"24 what?"

"Triangles."

"How many squares do you have?"

Puts 4 fingers on triangles in each new unit and counts each square: "6!"



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Learning Trajectories

- Mathematics *of* children— *representations* and thinking of children as it developments naturally
- Activities *matched* to children's development in each topic
- Therefore:
  - All within developmental capacities of children
  - Provide a natural "building block" to the next level
  - Provides *mathematical* building blocks for school success

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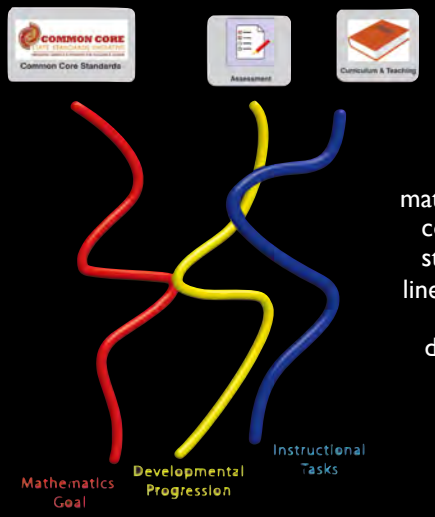


Present day, research on mathematics goals contributes to standards (red line) and so forth, but… disconnected.

6

Scientific Approach to Learning Trajectories weaves the 3 parts together

7

Why Might *You* Care?

- How might *you* use LTs?
  - Keeping track
  - Checking up
  - Finding out
- Perhaps most powerful, underused teaching strategy: Formative assessment

8

The Quality of Children's Experiences

- 16 teachers of 6, 7-year-olds, considered above average, in the U.K. Dedicated, conscientious.
- More than 1/2 of tasks were mismatched
  - misdiagnosis
  - failures in task design

9

The Quality of Children's Experiences

- High attainers were underestimated—41%
  - never even recognized that they underestimated
  - in *no* case was a task considered too easy
- 80% more practice tasks than intended
- children were "cheerful and industrious" and didn't mind doing the same old work

10

The Quality of Children's Experiences

- Low attainers overestimated—44%
- Only 1/9 of time moved to a lower level
- Most of the time, just moved to next set of tasks

11

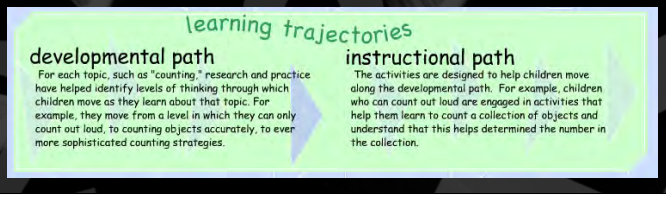
Formative Assessment

- Increases achievement more than most interventions
- Teachers' assessments "have effect sizes from .4 to .7 standard deviations, larger than most effects of instructional programs, which are considered impressive with a .25 effect size" (Lorrie Shepard).

12

Learning Trajectories: 3 Parts

1. Goal
2. Developmental Progression
3. Instructional Activities



13

Learning Trajectory for Counting

1st: Goal: Accurate, confident object counting

2nd: Developmental Progression…

14

Learning Trajectory for Counting

- *Precounter* Says number words, not sequence: "one, two, four…".
- *Chanter* Says in sequence but may run together
- *Reciter* Verbal counting to 5, then 10

15

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17

Learning Trajectory for Counting

- *Corresponder* Counts correctly using 1-1 correspondence, *at least up to 5 objects in a line.*

18

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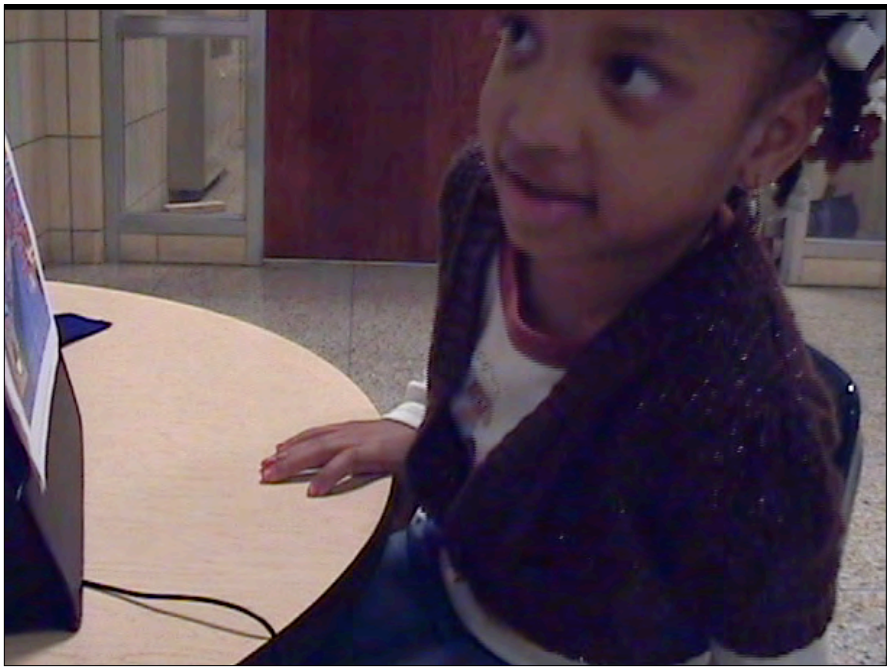

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Learning Trajectory for Counting

- *Corresponder* Counts correctly using 1-1 correspondence, *at least up to 5 objects in a line*
- *Counter (Small Numbers)* Counts 1-5 objects in a line meaningfully (i.e., employ the cardinal rule)

21

Learning Trajectory for Counting

- *Producer (Small Numbers)* Counts *out* a collection up to 5

22

23

Learning Trajectory for Counting

- *Producer (Small Numbers)* Counts *out* a collection up to 5
- *Counter (10)* Counts collections up to 10
- *Counter and Producer (10+)* and keeps track of unorganized collections

24

Learning Trajectory for Counting

- Counter from N

25

26

Learning Trajectory for Counting

- Counter from N
- **Counter On Using Patterns**
- Counter On Keeping Track
- Counter Forward and Back

27

Poll #1

28

29

What level of thinking?

1. Chanter
2. Reciter
3. Corresponder
4. Counter (Small Numbers)
5. Producer (Small Numbers)

30

Poll #2

31

32

What level of thinking?

1. Chanter
2. Reciter
3. Corresponder
4. Counter (Small Numbers)
5. Producer (Small Numbers)

33

Instructional Activities: *3rd* Part of Learning Trajectories

34

Small Numbers and Counting

- Finger plays:
  - When I was one…

    When I was one, I was so small, (hold up 1 finger)
    I could not speak a word at all. (shake head)
    When I was two, I learned to talk. (hold up 2 fingers)
    I learned to sing, I learned to walk. (point to mouth and feet)
    When I was three, I grew and grew. (hold up 3 fingers)
    Now I am four and so are you! (hold up 4 fingers)
  - Later: Five Little Monkeys, etc.

35

Count and Move

36

Counting Circle

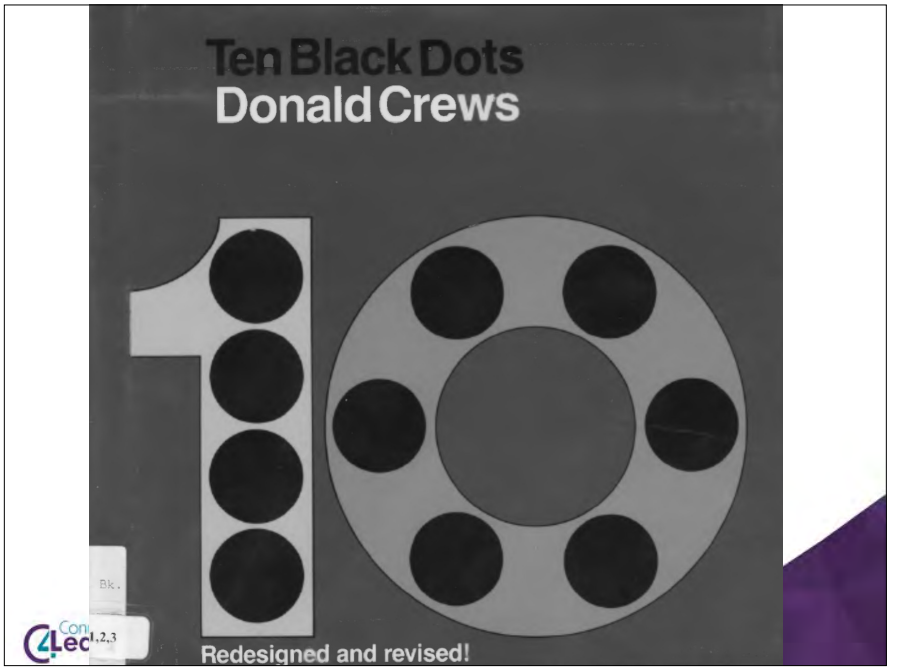
37

38

Books Limited

- 68% include numbers less than or equal to 10
- Only 12% present the number 0 in comparison to 90 percent of the books that presented the number 1.
- Less than ½ present 3 representations (numeral, number word, and quantity)

39



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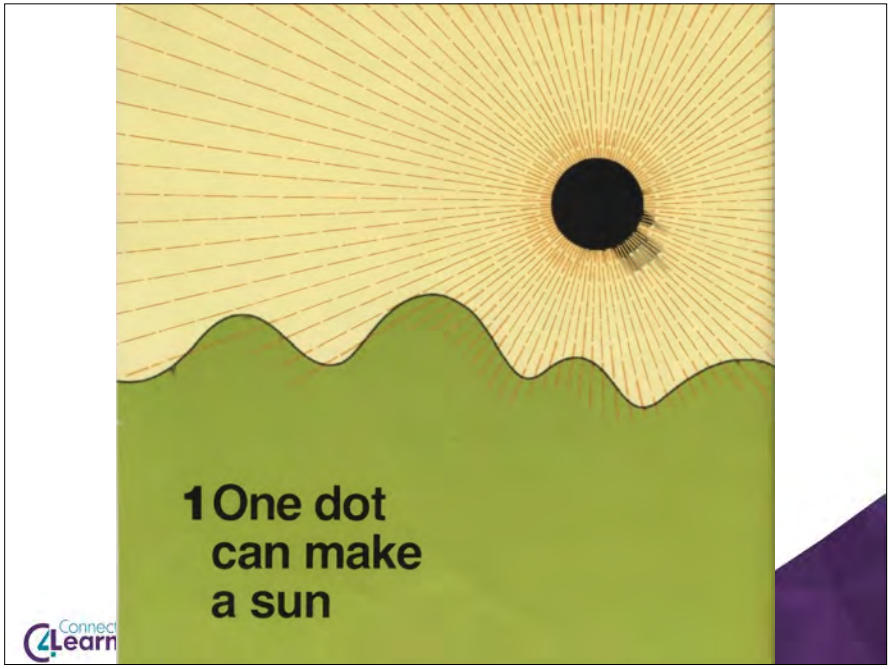
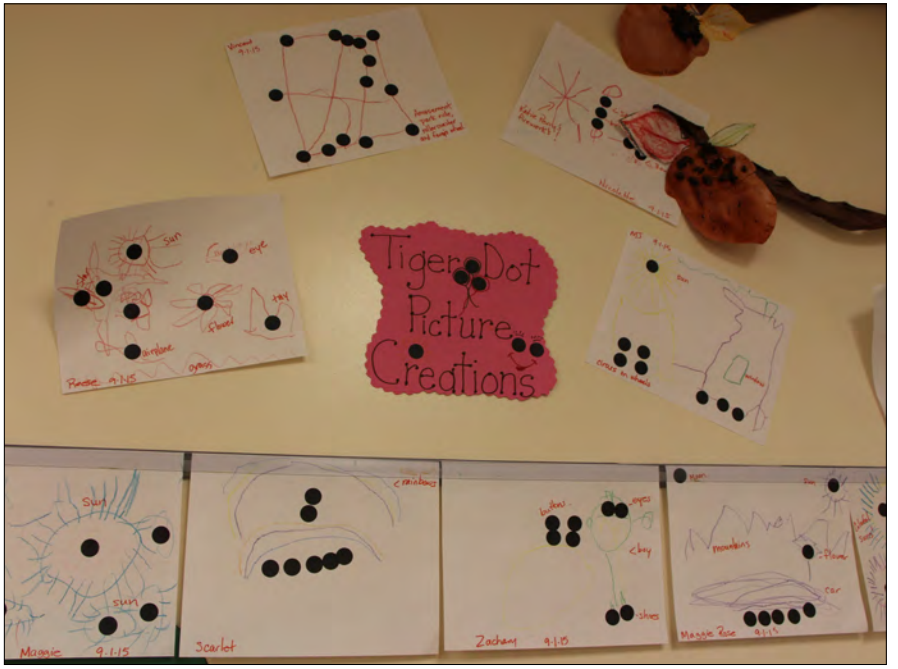
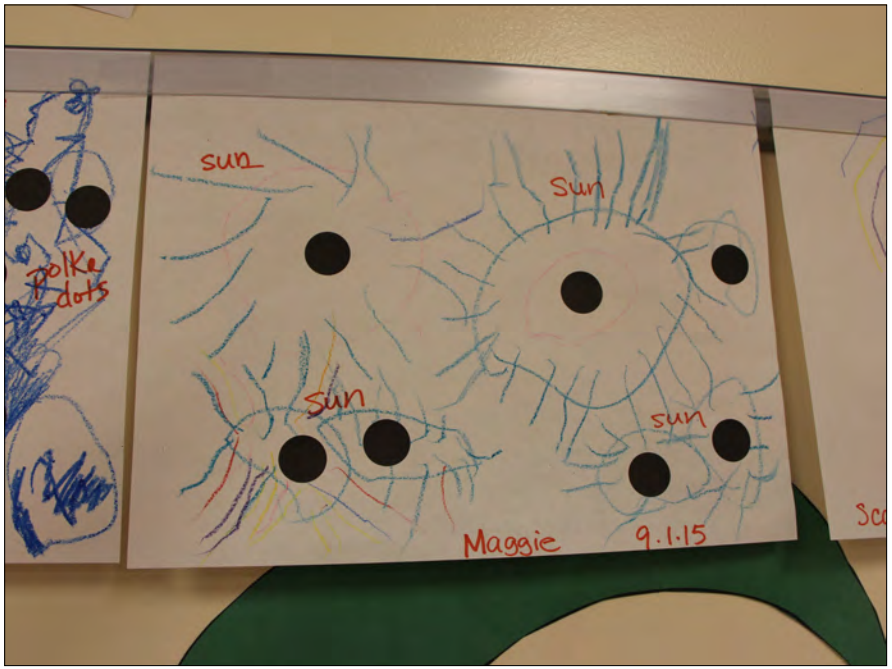

1 One dot
can make
a sun

41


Tiger Dot
Picture
Creations

42


sun
Sun
sun
sun
polka dots
Maggie
9.1.15

43


Brycen
Bird
Crocodile
sun
polka dots
car
9.1.15

44

Simon Says

45

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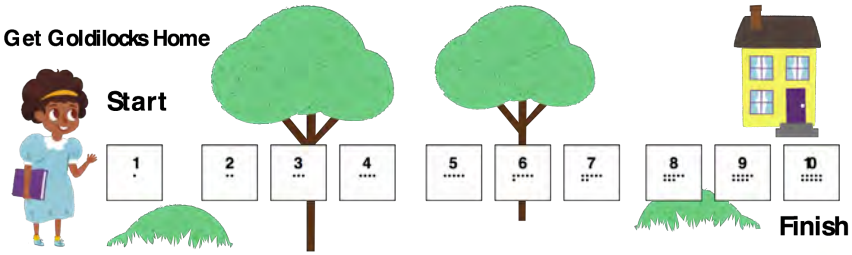
Path Game…and Beyond



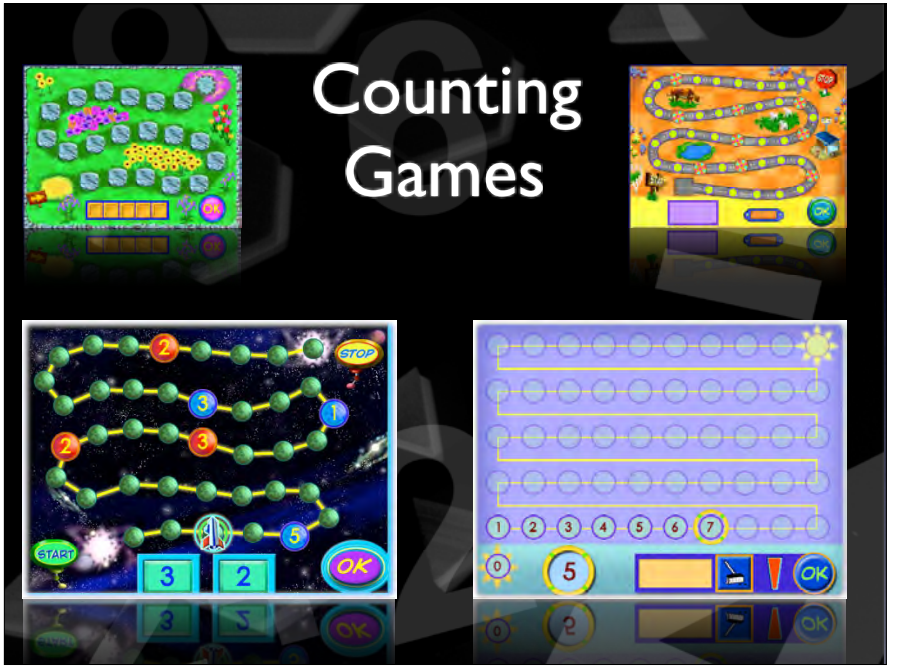
48

What Number Now?

Connect 4 Learning

49

Counting Games



50

51

Road Race: Connecting Representations

- Count the dots and move that number of jumps
- *Connecting* different representa-tions of number!

52

Road Race Shape Counting - Another Variation

- Count the sides of a shape and move that number of jumps
- Connecting new concepts of number

53

Poll #3

54

What level of thinking is this *teaching/practicing*?

1. Chanter
2. Reciter
3. Corresponder
4. Counter (Small Numbers)
5. Producer (Small Numbers)

55

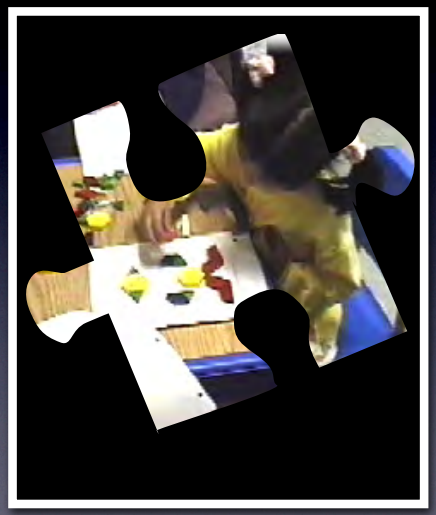
Space Race Number Choice

- Choose the "better" of two numbers
- Comparing but also reasoning: Which is better in this case?

56

57

A Trajectory for Composing Geometric Shapes



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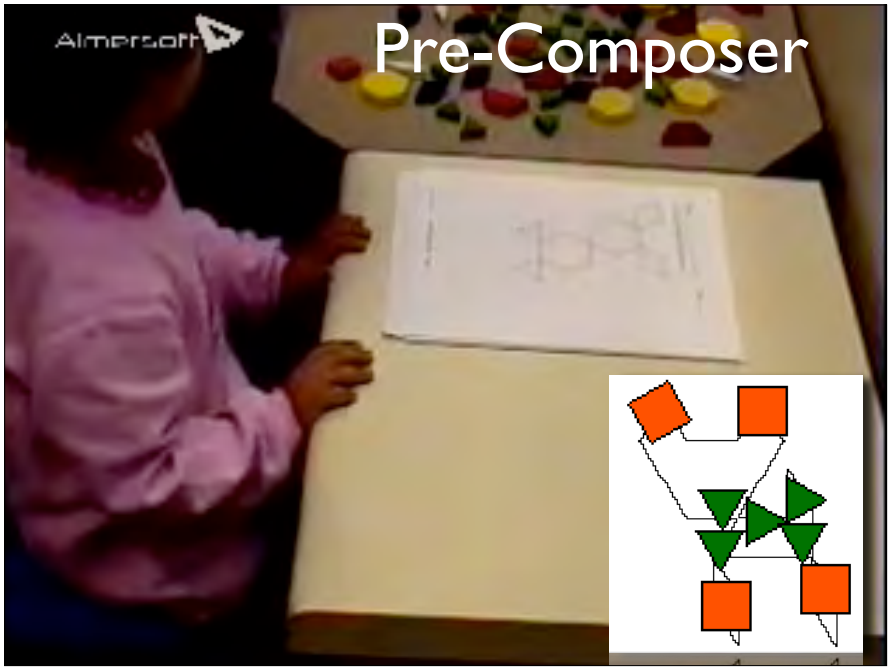

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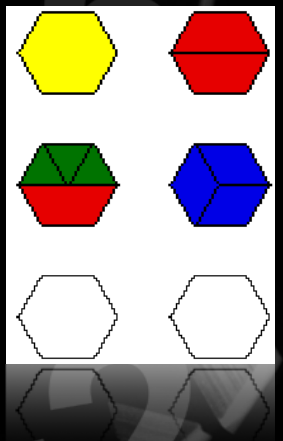
60

Shape Composer

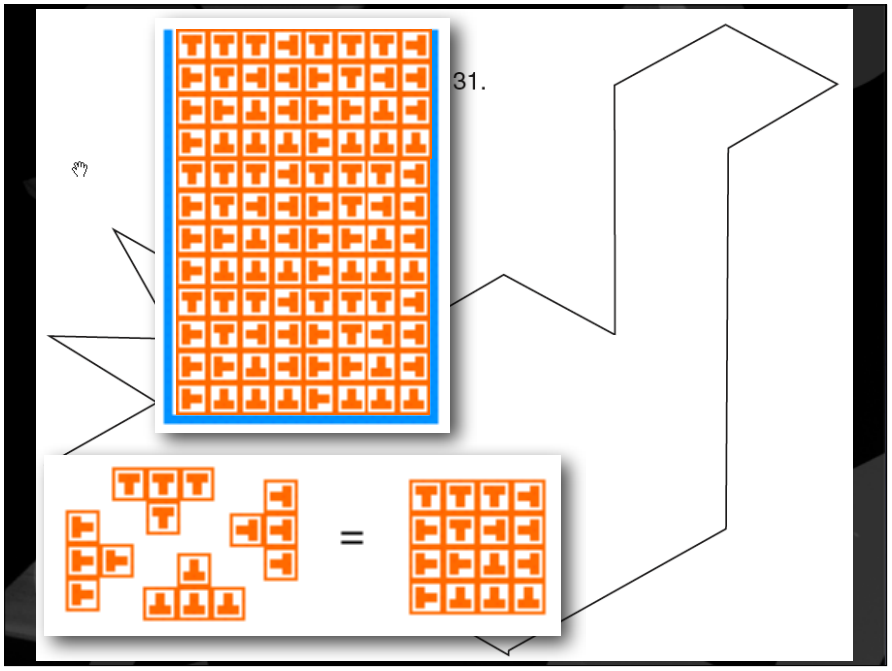
61

Substitution Composer



- Finds different ways to fill a frame, emphasizing substitution relationships.

62



63

Create A Scene

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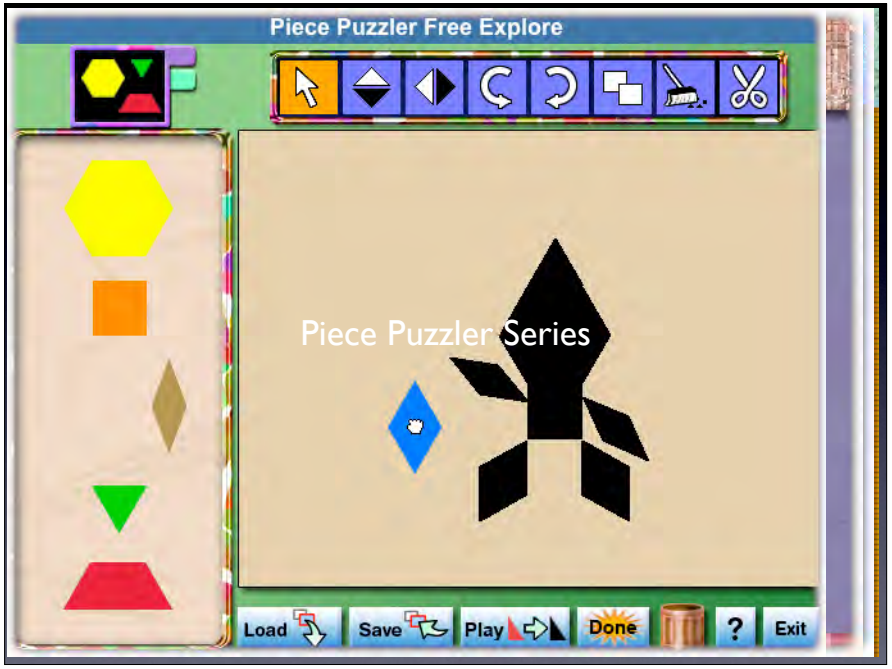
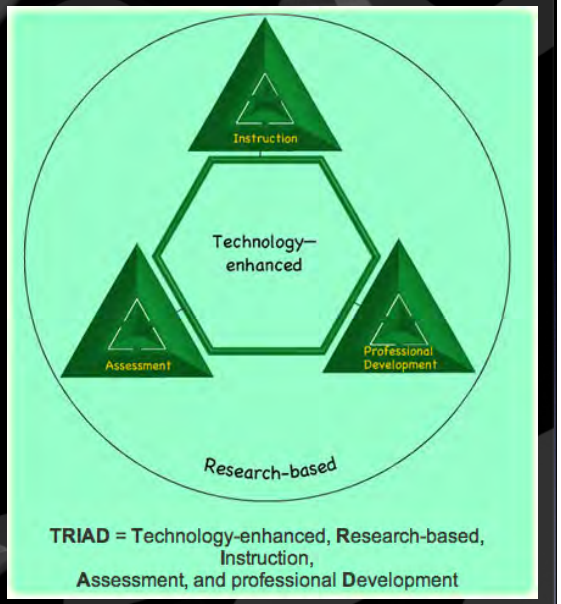


65

TRIAD II: Large-Scale Evaluation

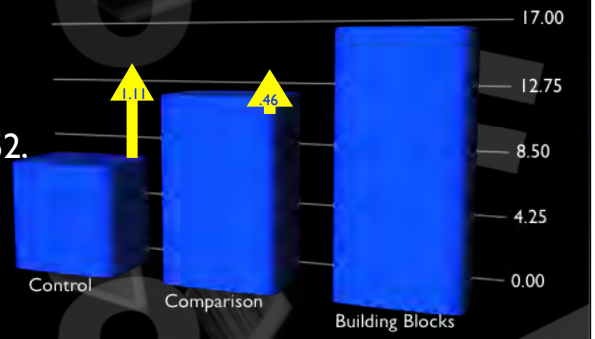


TRIAD = Technology-enhanced, **R**esearch-based, **I**nstruction, **A**ssessment, and professional **D**evelopment

66

Results: Child Assessment

- F(1, 32) = 40.52. p = .000+
- T Scores:
  - 50 Mean
  - 10 SD



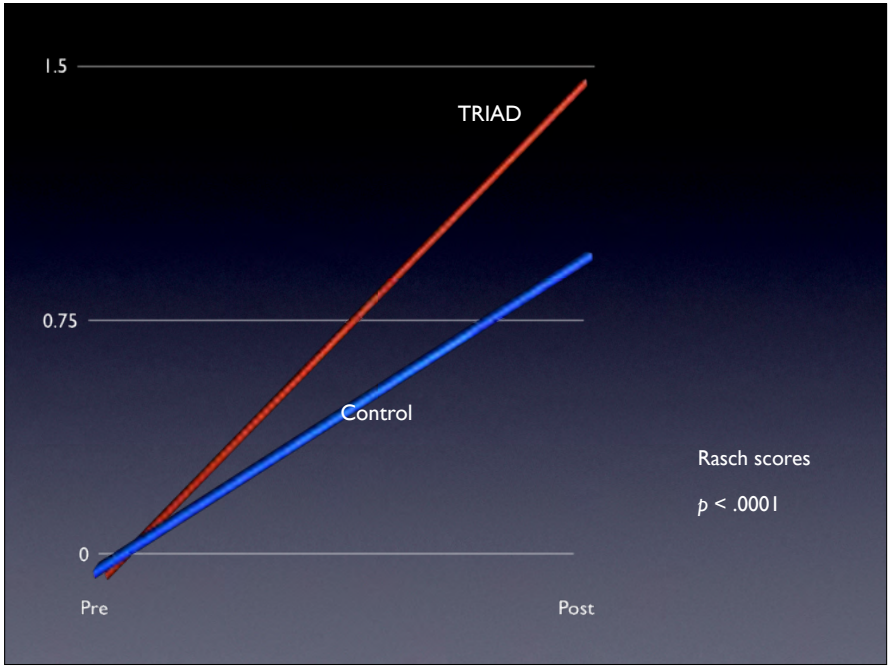
Clements, D. H., & Sarama, J. (2008). Experimental evaluation of the effects of a research-based preschool mathematics curriculum. *American Educational Research Journal*, *45*, 443-494.

67

Building Blocks In the News

- http://www.thebostonchannel.com/news/15776035/detail.html

68



69

Using the Learning Trajectories

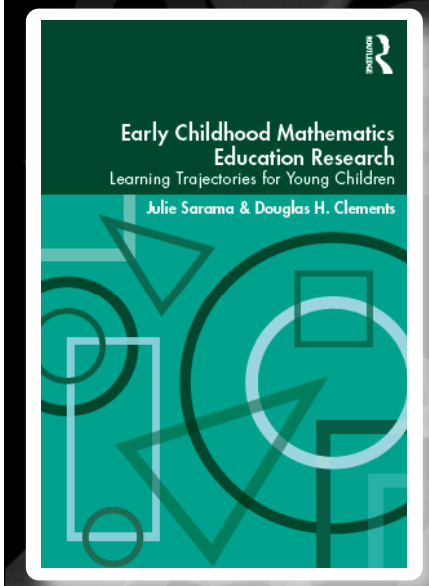
It takes time. A teacher talks about interviewing a child for report cards:

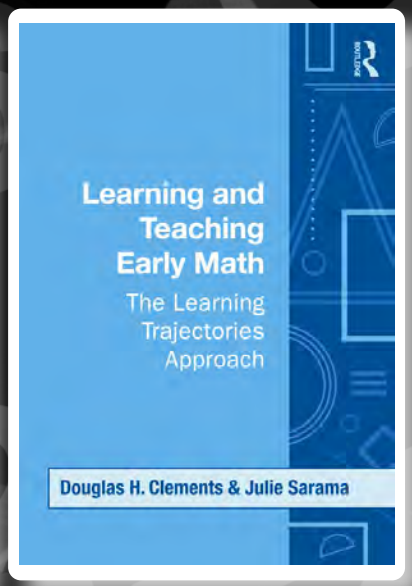
"She was able to do verbal counting to 8, and then when she slowed down, she could get to 11. So I said, "Can you make me a group of 6?" And so she did. So then I added, I did 12, I think. She couldn't do it.

Then I noted that, *so now I'm thinking in the trajectories*, I think she's a "Counter (Small Numbers)," right? She's on her way to being a "Counter (10)." She's in between the two. So that's what I was thinking of as I did this."

—Pat, 2004

70





71

Web Sites and Contact

Connect4Learning.com

TRIADscaleup.org

"If we teach today as we taught yesterday, we rob our children of tomorrow." –John Dewey

72